Algorithm of decoding of convolutional codes in communication links with multiplexing

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I. INTRODUCTION

The perspective direction of increase in reliability of reception of messages from the information transmission systems (ITS) is development of the new models, methods and techniques allowing to lift the known limits in the form of the uniform distribution law of probabilities (DP) of characters of the transferred message and the binomial distribution law of probabilities of elements of a flow of errors in the discrete communication link. So, the multiplex digital flows (MDF) often have the redundancy caused by existence of the regular sequences (RS). Besides, in some cases in the discrete communication link (DCL) SCS at the sources of errors (SE) Markov properties are shown [1, 2].

In satellite communication systems (SCS) application of the noiseproof convolutional codes (CC) is marked wide (more than 40%). In practice for decoding of CC the greatest distribution was gained by the algorithm of Viterbi realizing criterion of maximum likelihood (ML). As the majority of high-speed digital communication links (HSDCL) in SCS (more than 90% of HSDCL), represent MDF at which there is a redundancy caused by existence of DP at estimation of statistical characteristics of SE it is required to solve problems of search and separation of such sequences, receiving a flow of errors and computation of statistical characteristics of SE on a CC decoder input.

At the same time it is necessary to consider conversions of messages in an information transmission path (the additive scrambling (AS), multiplexing etc.), using the DCL model with additive discrete noise.

Most channels SCS has binomial distribution of probabilities of errors or the close to binomial. However in some cases, described in [1,2], the nature of error distribution in DCL SCS is excellent from binomial, owing to different hindering

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influences. As a result SE in DCL have Markov properties for which accounting the DCL models on the basis of the Markov chains (MC) are used. The appropriate accounting of real statistical characteristics of SE in algorithms of decoding of noiseproof codes allows to increase reliability of reception of messages. Thus, it is required to solve the problem consisting in development of a method of estimation of statistical characteristics of SE and sources of messages (SM) on the basis of prior information concerning structure of MDF and an algorithm of decoding of CC providing lowering of value of probability of an error on the basis of use of estimates of distributions of probabilities of SE and the SM, having an appearance:

Initial data: a digital flow with convolutional coding:

\[ Y(t) = \theta(\Phi(A(t))) \oplus E(t) \]  

(1)

\( A(t) \) – multiplexing digital flow; \( \theta \) – the operator describing the rule of convolutional coding; \( \Phi \) – the operator describing the rule of scrambling; \( E(t) \) – flow of errors; \( t \) – the discrete time; \( \oplus \) – addition on the module 2.

It is necessary: on the basis of prior information concerning structure of MDF, rules \( \theta \) and \( \Phi \) to overcome prior uncertainty concerning distributions of probabilities of SM \( \{p(e_j)\} \) and condensed in MDF SM \( \{p(a_i)\} \) and to develop the algorithm of decoding of the convolutional code providing

\[ P_0 \Rightarrow P_{0\text{ min}} \]  

(2)

where \( P_0 \) – probability of an error on bit on a decoder output, \( P_{0\text{ min}} \) – the minimum probability of an error on bit on a decoder output, with the available accuracy of prior these rather statistical characteristics. At the same time the following restrictions are accepted: in ITS the AC is used; conditions of reception allow to provide reliable clock and frame alignment; the structure of MDF does not depend on time; prior uncertainty concerning structure of MDF and the scrambling sequence is absent; digital flows of the condensed SM and SE represent stationary accidental processes. The structure of such information transmission system is provided in the Figure 1.

Figure 1. The structure of such information transmission system with multiplexing and chain coding

Let's consider a mathematical model of the digital flow with convolutional coding accounting of structure of MDF, statistical properties the multiplexed SM and SE. The DCL statistical properties, formed by set of devices between a set condensed in MDF SM and a CC decoder input, depend on structure of MDF which is described by a multiplexing template in a look

\[ A(t') = S\left[\{A^h(t)\}_{h=1, N}\right] \]  

(3)

where \( S \) – the function reflecting the rule of fixing of bits in cycle MDF for condensed by the IC; \( t' \) – sequence number of bit in cycle MDF; \( A^h(t) \) – implementation of the message of \( h \) SM.

As CC put into practice are constructed on the basis of CC with a code \( R \) speed = 1/2, are of interest the distribution of final probabilities of a two-dimensional binary random variable in a flow of \( \langle h \rangle \) SM determined by Markov's formula

\[ p \left( a^h_{i-1} a^h_i \right) = p \left( a^h_{i-1} \right) p \left( a^h_i / a^h_{i-1} \right) \]  

(4)

where \( a^h_i \) –character \( i \) of SM \( h \); \( p(a^h_{i}/a^h_{i-1}) \) – the transition probability of SM \( h \).

Realization of the message of \( h \) SM on length of cycle MDF is described by expression

\[ \{A^h(t)\} = \{a^h_1, a^h_2, ..., a^h_{N_h}\} \]  

(5)

where \( h \) – sequence number of the SM in MDF; \( N \) – the number of the SM condensed in MDF; \( N_h \) – the number of bits of \( h \) source on cycle
MDF length; \( t \) – sequence number of bit of the SM. Here in after the message of the SM is understood as the message on an SM coder output.

Taking into account structure of static MDF, the summary digital flow on an output of the multiplexer is represented expression

\[
A(t') = (a_i^1(1), a_i^2(2), \ldots, a_i^{N_{c}}(mN_{c}))
\]

where \( a_i^h(t) \) – \( i \) the character in the message of \( h \) SM located on a position in MDF \( N_{c} \) – cycle MDF length.

Redundancy of the block message depends as on a compression ratio of messages in the condensed channels, and on value of utilization coefficient of throughput of MDF defined by expression

\[
R_{MDF} = \frac{1}{N_{c}} \sum_{h=1}^{N} s_h
\]

where \( s_h \) equal 0 in the absence of payload in the \( h \) SM MDF and equal to number of bits of the selected \( h \) SM in case of its existence, \( N \) – the number of the SM condensed in MDF.

Taking into account structure of MDF and correlative communications between characters of messages in the condensed channels, probability of value the \( t \) bit in MDF \( A(t) \) selected for transmission of information the \( h \) SM, is defined by expression

\[
p\left(a_i^h(t')\right) = p\left(a_i^h(j), \ldots, a_i^h(j')\right) p(a_i^{h+1}(t)/a_i^{h-1}(j), \ldots a_i^h(j'))
\]

where \( j, j' \) - sequence numbers of bits of MDF selected for the \( h \) SM just before - \( m \) bit; \( i \) - sequence number of bit in the message of \( h \) SM; \( n \) – connectivity of Markov Chain.

Further, according to the model provided in the Figure 1, MDF, the entering on an input, single-digit defines the sequence on his output

\[
B(t) = A(t') \oplus M(t)
\]

where \( M(t) \) - pseudorandom sequence (PRS).

Then taking into account the known PRS \( M(t) \), for \( t \) of bit of the sequence value of probability of bit of the sequence from an output of AS

\[
p(b(t')) = p(a(t') \oplus m(l))
\]

where \( m(l) \) – bit \( l \) of PRS \( M(t) \).

The probability of zero on output of the CC coder is defined by probability of the binary combinations having zero or even value of weight of Hamming, formed by values of bits in the cells of the register of shift connected on an adder input.

As now the rule of specifying of subsets and binary combinations of arbitrary length which amount of elements is equal 0 and 1 respectively does not exist, it is offered to determine them by recurrent expressions

\[
s(i) = s'\left(i - 2^{ \log_2{j} }\right) + 2^{ \log_2{j}+1 }
\]

\[
s'(i) = s\left(i - 2^{ \log_2{j} }\right) + 2^{ \log_2{j}+1 }
\]

where \( i \) – decimal sequence number of an element in a subset, \( s'(0) = 0 \) and \( s'(0) = 1 \) – the first element of subsets \( S \) and \( S' \) respectively.

Taking into account probabilities of bits on an output of the scrambler and the received recurrent expressions, the probability of bit on output of the register of shift of CC with a speed \( R=1/2 \) located on a position \( t' = i' / R = 2i' \) is defined by expressions

\[
p(x(2t') = 0) = \sum_{i=0}^{2^{v-1} - 1} \prod_{v=0}^{v_{\max}} p(b(t'+v) = BIN(j = s(i), v))
\]

\[
p(x(2t') = 1) = 1 - p(x(2t') = 0) = \sum_{i=0}^{2^{v-1} - 1} \prod_{v=0}^{v_{\max}} p(b(t'+v) = BIN(j = s'(i), v))
\]

where \( j \) is decimal representation of a binary number; \( BIN(j, v) \) – \( v \) value of bit in binary representation of a decimal number of \( j \); \( v_{\max} \) – the maximum value of number of a cell of the register of shift of CC connected to the adder; \( p\left(x(2t')\right) \) – probability of bit on CC output; \( v' \) is number of leadouts of the register of shift of CC connected to the adder.
We will present noises in the form of a binary combination of a source of errors with length of \( n \) of characters from a set
\[
E = \{ e_s \}, s = 0, ..., 2^n - 1
\]
with DP set on it
\[
p(E) = \{ p(e_0), p(e_1), ..., p(e_{2^n-1}) \}. \tag{15}
\]
As a result on an input of the decoder length combinations from a set arrive
\[
Y = \{ y_0, y_1, ..., y_{2^n-1} \}
\]
with DP
\[
p(Y) = \{ p(y_0), p(y_1), ..., p(y_{2^n-1}) \}. \tag{16}
\]
For CC with \( R = 1/2 \), \( n = 2 \), and probabilities of equality to zero even and odd bits in an output on an input of the SK decoder are defined by expressions
\[
p(y(2t') = 0) = \sum_{z=0}^{1} p(x(2t') = z) p(e(2t') = z)
\]
\[
p(y(2t'+1) = 0) = \sum_{z=0}^{1} p(x(2t'+1) = z) p(e(2t'+1) = z)
\]
where \( p\left(y(2t')\right)\) – probability of bit on a CC decoder input; \( p\left(e(2t')\right)\) – probability of a bit error, \( z \in \{0,1\} \).

As each combination of a set of \( Y \) is result of addition on the module 2 resolved code combinations \( x_i \) with certain \( e_s \), the probability of an output is defined in a look
\[
p(y(2t') = z_o, y(2t'+1) = z_i) = p(y(2t') = z_o) p(y(2t'+1) = z_i) \tag{17}
\]
Probabilities an output \( y(2t') = z_o, y(2t'+1) = z_i \) are defined by system of the equations
\[
\begin{alignat}{2}
\quad \quad & p(y_0) = \sum_{i=0}^{3} p(x_i) p(e_s = y_0 \oplus x_i) \\
\quad & p(y_1) = \sum_{i=0}^{3} p(x_i) p(e_s = y_1 \oplus x_i) \tag{18} \\
\quad & p(y_2) = \sum_{i=0}^{3} p(x_i) p(e_s = y_2 \oplus x_i) \\
\quad & p(y_3) = \sum_{i=0}^{3} p(x_i) p(e_s = y_3 \oplus x_i)
\end{alignat}
\]
where, \( x_i, y_i = \{00,01,10,11\}, i = 0, ..., 3 \) and to timepoint \( 2t' \) there corresponds the first bit of the output, and \( 2t'+1 \) – the second bit of the output.

In case of binomial DCL distribution of probabilities is defined by Bernoulli's formula. With the SM Markov properties the mathematical model of binary random process provided in [7] is used. Last allows to define distribution of final probabilities of the two-dimensional binary random variable (BRV) \( p(00), p(01), p(10), p(11) \), the set single-connected Markov Chain, on the basis of estimation of distribution of final probabilities of one-dimensional BRV \( p(0) \). As the case when probability errors more than 0,5 is of practical interest, calculation of probabilities of two-dimensional BRV of SE is defined by expressions:
\[
\hat{p}(00) = \frac{3p(0) - 1}{2}; \\
\hat{p}(01) = \hat{p}(10) = p(0) - \hat{p}(00); \tag{19} \\
\hat{p}(11) = p(1) - \hat{p}(10)
\]

II. ESTIMATED RESULTS

The analytical model provided by set of expressions (3–19) define DP on sets of the binary sequences on inputs and outputs of the conversion devices of the message which are DCL part.

Information on statistical characteristics of a flow of errors can be received on the basis of the analysis of regular sequences (RS) in MDF. Such information can be obtained in case of sequential demultiplexing of a flow of each condensed channel, assessment of its period, demultiplexing of subflows from the flow condensed with the SM on earlier evaluated period. RP is accepted the equal combination transferred on the estimated value of the period.

After restoration of a share of information estimation of DP of combinations of SE on the basis of a binomial model or expression (19) is carried out. At the same time as \( p(0) \) value the occurrence of zero in the selection of a flow of errors, available to the analysis, received by
addition on the module 2 of the flow decoder watched on an input with regenerated is used.

For estimation of DP of combinations of CC corresponding to flows on an output of the SM condensed in MDF the method based on the solution of the linear equation system (LES) provided by expression (18), Gauss’s method is offered.

In the beginning it is required to estimate probabilities of two-dimensional DRV in a flow of errors. Expression (19) defines values of estimates of two-dimensional DRV on the basis of the value \( \hat{p}(0) \) possessing an error \( \Delta \hat{p}(0) \) concerning the true value. Therefore, the accuracy of estimation of probability of two-dimensional DRV depends on the accuracy of estimation of one-dimensional DRV and changes according to Table 1.

Table 1. The Accuracy of estimation of probability of a two-dimensional binary random variable, in case of \( \hat{p}(0) > 0.5 \)

<table>
<thead>
<tr>
<th>( \hat{p}(e = 00) )</th>
<th>( \hat{p}(e = 01) )</th>
<th>( \hat{p}(e = 10) )</th>
<th>( \hat{p}(e = 11) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+3\Delta \hat{p}(0)/2)</td>
<td>(-\Delta \hat{p}(0)/2)</td>
<td>(-\Delta \hat{p}(0)/2)</td>
<td>(-\Delta \hat{p}(0)/2)</td>
</tr>
</tbody>
</table>

At the same time the accuracy of estimation \( \hat{p}(0) \) depends on the volume of selection of \( N \) of the parameter \( \hat{p}(0) \), necessary for estimation, with the required values of reliability \( \beta \hat{p}(0) \) and accuracy \( \Delta \hat{p}(0) \). Further estimation of probabilities of two-dimensional DRV in a flow on CC output is executed. At the same time estimates of probabilities of two-dimensional DRV are calculated separately for subflows of CC corresponding to flows in each condensed SM. Estimation error \( \{p(e_i)\} \), where \( e_i \) — outputs in a flow of errors are defined by perturbation of LES (18). Dependence of accuracies of estimation of probabilities of two-dimensional DRV on an output of CC \( \Delta \hat{p}(x_i) \) and one-dimensional DRV \( \Delta \hat{p}(0) \) in a flow of errors is provided in the Figure 2.

The received DP estimates an output are used in a decoding algorithm. Probabilities of edges of a grid are defined by expression

\[
\hat{p}'(K) = \hat{p}_b \hat{p}(K')
\]

where \( \hat{p}(K') \) — probability of appearance of the sequence \( p(x_i) \) on a coder output; \( \hat{p}(e_s) = p(y_d \oplus x_i) \) — probability of a combination of an error \( e_s \). The probability of the way passing through an edge in a node \( K \) on a grid of CC is defined by expression

\[
p(K) = \sum_{K=K1}^{K2} p'(K)
\]

where \( p(K') \) — probability of passing of a way through node of a grid on the previous tier. The rule of a choice of the survived way (SW) from two, is defined by expression

\[
K = \arg \max_{K=K1}^{K2} \sum_{K=K1}^{K2} p'(K)
\]

Where \( K_1, K_2 \) — nodes of a grid of CC from which transition to a node \( K \) is possible. The rule of a choice of SW carrying in each of nodes on a step \( \tau \) is defined by expression

\[
K_{opt} = \arg \max_{K=0}^{2^\tau-1} \sum_{K=0}^{2^\tau-1} p'(K)
\]

Information on the sequences of the encoded characters recovered by the decoder \( \hat{x}_t = (\hat{x}_{t,0}, \cdots, \hat{x}_{t,\tau-1}) \) corresponding to SW with a depth \( \tau \) for \( t \) the initial moments of creation of the ways provided in the Figure 3 is not considered.
The essence of a method consists in computation on the basis \( \{\hat{X}_t\} \) of estimates of probabilities an output on a \( t \)-position of error \( \hat{p}(c_i, t) c_i \in \{00, 01, 10, 11\} \) according to expression

\[
\hat{p}(c_i, t) = \frac{1}{\tau} \sum_{j=0}^{\tau} \sum_{j=\tau-t}^n f(\hat{e}_{t-j, j}, c_j), \quad \hat{e}_{t-j, j} = y_j \oplus \hat{X}_{t-j, j}.
\]

(26)

where \( \tau \) – decoding depth, \( \hat{e}_{t-j, j} \) – estimate of an output of a flow of errors on a position \( t-j \) in SW \( j \); \( y_j \) – an output of a digital flow on a decoder input; \( \hat{X}_{t-j, j} \) – an output on a position \( t-j \) in SW \( j \); \( f(\hat{e}_{t-j, j}, c_j) \) – the indicating function accepting value 1 (0) in case of equality (inequality) an output in SW.

\[
f(\hat{e}_{t-j, j}, c_j) = \begin{cases} 1, & \text{if } \hat{e}_{t-j, j} = c_j \\ 0, & \text{if } \hat{e}_{t-j, j} \oplus c_j \end{cases}
\]

(27)

The offered method allows to estimate reliabilities of the decisions received in the CC decoder on the basis of statistical information processing about SW. The received estimates are used further as input data at the following stage of decoding by criterion of the MPP.
III. PROPOSAL ALGORITHM AND SIMULATION RESULTS

Thus in total the offered methods of estimation of DP of SM and SE allowed to offer the algorithm of decoding of CC in communication links with multiplexing considering structural and the DCL statistical properties, provided in a the Figure 4.

As in practice, as a rule, there is no information concerning existence of correlative communications in a flow of errors, in an algorithm the DCL model identity burst on the basis of comparing of quality of the decoding provided with algorithms of ML and MPP on RS MDF sections is provided.

In a Figure 5 the diagram of MPP1 characterizes results of decoding on the basis of the analysis of statistical characteristics of DCL on RS MDF, the diagram of MPP2 characterizes results of decoding taking into account a method of posteriori estimation of the statistical characteristics of SE which are dynamically changing in time, based on information analysis, received from the MPP1 decoder.

IV. CONCLUSION

The paper proposed the threshold and probability estimation presented in Section III for good efficiency in solver, convolution coding according to the flowchart of Figure 4. This algorithm is capable of being applied to digital telecommunications systems.

REFERENCES


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